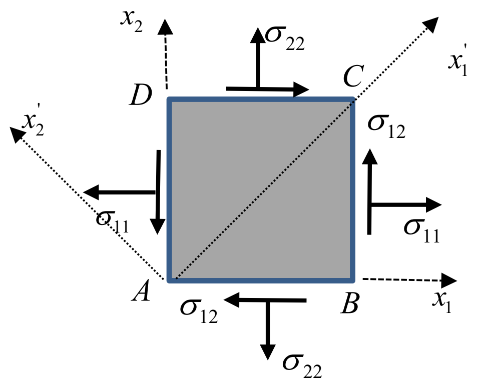
Exercise 1: A 50 mm thin square aluminum plate (Young's modulus, $E = 70GPa$ Poisson's ration v=0.3) is subjected to the stresses shown in the figure below. Calculate the change in length, of the diagonal *BD* in two ways:

(a) Determine the strains with respect to x_1, x_2 and employ the strain transformation equation.

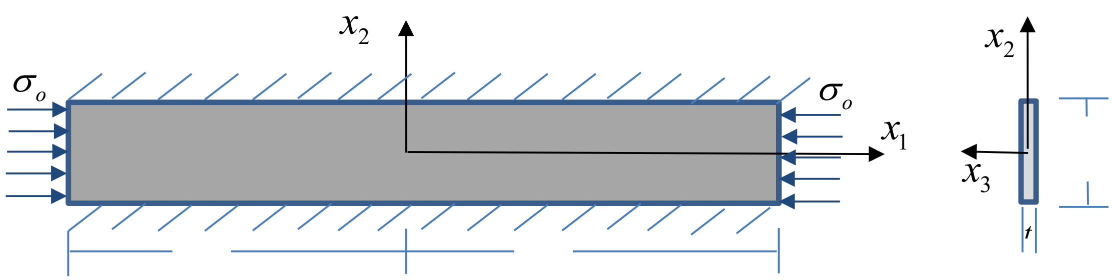
(b) Determine the stresses with respect to x_1', x_2' and use the Hook's law.



$$\sigma_{11} = 20\text{MPa}$$
$$\sigma_{22} = 10\text{MPa}$$
$$\sigma_{12} = \sigma_{21} = 5\text{MPa}$$

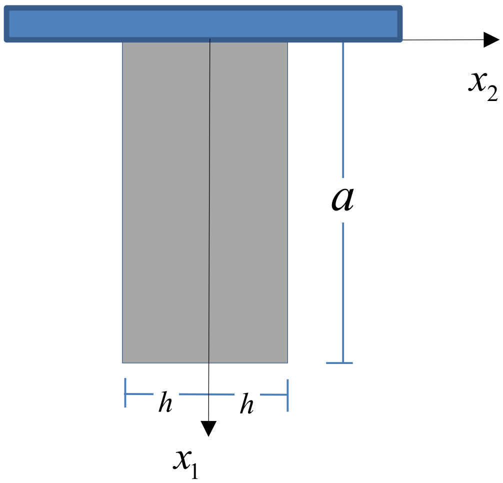
Exercise 2: The thin plate shown in the Figure is subjected to loads that produce the uniform stress at the two ends. The long edges are placed between two rigid walls. Show that the following given displacements are correct.

$$u_1 = -\frac{1-v^2}{E}\sigma_o x_1, \qquad u_2 = 0, \qquad u_3 = \frac{v(1+v)}{E}\sigma_o x_3$$



Hint: identify the state of stress and determine the stresses in the plate. Calculate the strains and integrate to obtain the displacements.

Exercise 3: A thin prismatic bar of specific weight γ and constant cross section hangs vertically (see Figure). Under the effect of its own weight, the displacement field is,

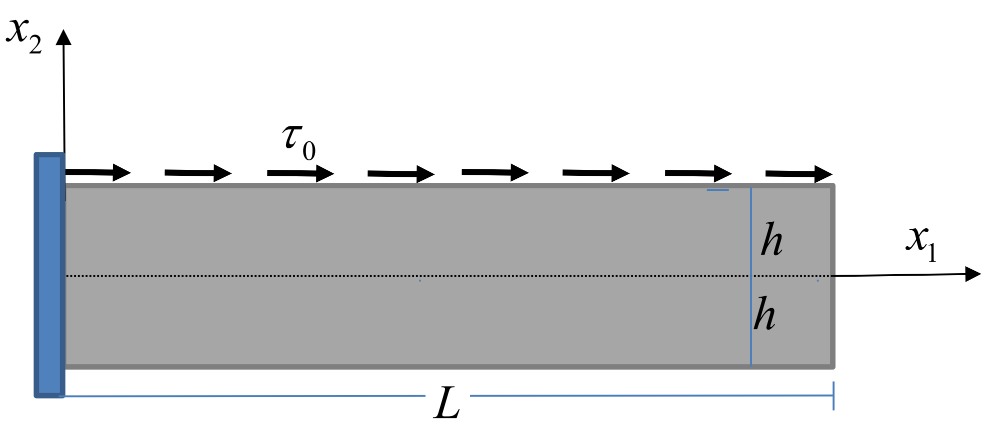


$$u_1(x_1,x_2) = \frac{\gamma}{2E}\left(2x_1 a - x_1^2 - \nu x_2^2\right)$$

$$u_2(x_1,x_2) = -\frac{\nu\gamma}{E}\left(a - x_1\right)x_2$$

Calculate the strain and stress components in the bar and check the boundary conditions if they are verified. The elastic properties of the bar (E,ν) are known and displacement and stress along the normal axis to the bar are neglected.

Exercise 4: The thin cantilever beam is subjected to a uniform shear stress along the entire upper surface as shown in the Figure.



Determine if the following Airy stress function is appropriate for this problem.

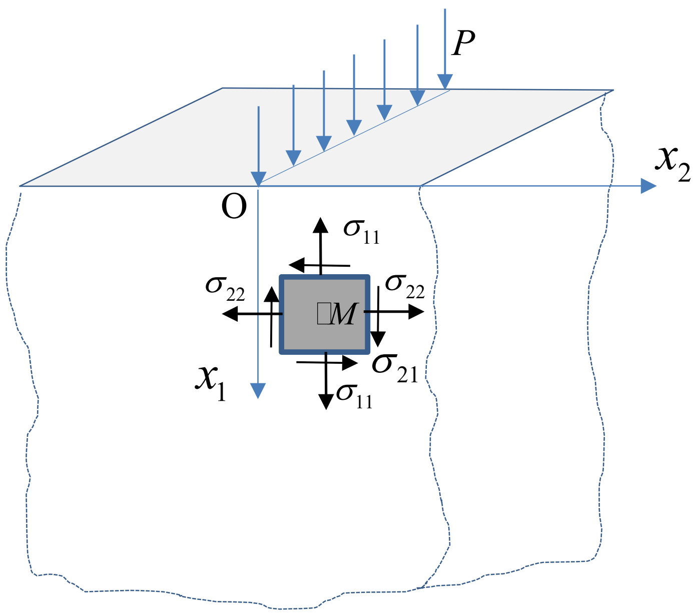
$$\Phi\left(x_1,x_2\right) = \frac{1}{4}\tau_0\left(x_1x_2 - \frac{x_1x_2^2}{h} - \frac{x_1x_2^3}{h^2} + \frac{Lx_2^2}{h} + \frac{Lx_2^3}{h^2}\right)$$

Exercise 5: For the line load on a semi-infinite body (see Figure below),

1: Show that the stress function,

$$\Phi(x_1, x_2) = -\frac{P}{\pi} x_2 \tan^{-1}\left(\frac{x_2}{x_1}\right)$$

results in the following stress field at a point *M* in the body.



$$\sigma_{11} = -\frac{2P}{\pi}\frac{x_1^3}{\left(x_1^2 + x_2^2\right)^2},$$

$$\sigma_{22} = -\frac{2P}{\pi}\frac{x_1 x_2^2}{\left(x_1^2 + x_2^2\right)^2},$$

$$\sigma_{12} = -\frac{2P}{\pi}\frac{x_2 x_1^2}{\left(x_1^2 + x_2^2\right)^2}$$

2: on a plane at a distance $x_1 = a$ consider the vertical sum of forces and show that equilibrium is satisfied (Hint: consider the vertical equilibrium of a stripe of unit thickness).

Note that: $\int_{-\infty}^{+\infty} \frac{a^3}{\left(a^3 + x_2^2\right)^2} dx_2 = \frac{\pi}{2}$

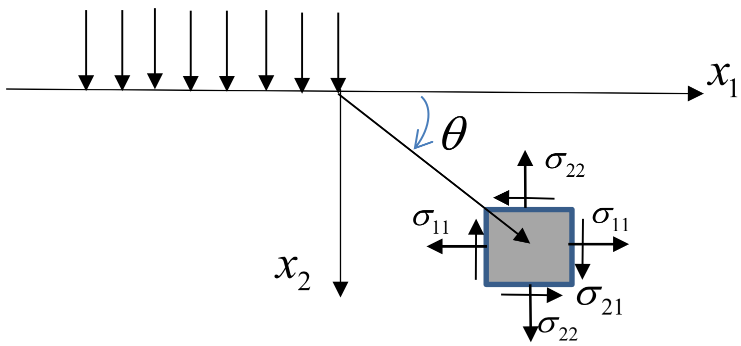
The following two problems were given in the first exam of the last year's course

Problem 1: The following stress function is given:

$$\Phi(x_1, x_2) = c\left[\left(x_1^2 + x_2^2\right)\tan^{-1}\frac{x_2}{x_1} - x_1 x_2\right]$$

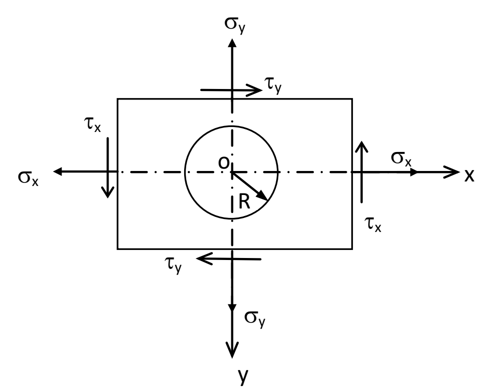
Show that:

1. it can provide a solution to the problem of an elastic semi-infinite medium subjected to pressure P on one side of the origin as shown in the Figure below.
2. calculate the corresponding stresses.
3. calculate the constant c.



Problem 2: At the center of a large thin plate, a circle of radius R is drawn with a pencil. Afterwards the plate is subjected to stresses as shown in the figure.

1. Demonstrate that the circle is transformed into an ellipse and calculate the difference between the semi-axes of the ellipse.
2. Determine the principal stresses.
   Data : Steel $E = 210GPa,\ \ v = 0.3$



$R = 10mm, \sigma_x = 300\text{MPa}, \tau_x = \tau_y = 150\text{MPa},\ \sigma_y = -100\text{MPa}$